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**ATIS Historical Foundations,  
Theory Development Updates, &  
ATIS-Analyses ...  
A Must-Read for Understanding *ATIS***

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Submitted as Part of the  
Maris M. Proffitt and Mary Higgins Proffitt Endowment Grants  
Indiana University  
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Bloomington, Indiana

# **A-GSBT Report**

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**The SimEd Basic Logic as Founded on the  
Logic of Axiomatic-General Systems Behavioral Theory:**

**A-GSBT and GST Perspectives**

Prepared by: Kenneth R. Thompson

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## ATIS Developments & ATIS-Analyses

### Abstract

This report provides developments of Thompson's *Axiomatic Theory of Intentional Systems (ATIS)* and *ATIS-Analyses*. Further, it provides an in-depth analysis of Theodore W. Frick's APT-analysis which is a temporal analysis of empirical events. Such an APT analysis is critical to the development of *ATIS* as it provides the basis for being able to analyze temporal empirical events by means of an axiomatic theory. Prior to this development, there had been no way to analyze temporal empirical events by an axiomatic theory, as such analyses have had to rely on statistical analyses. *ATIS*, founded on an emendation of *General Systems Theory*, provides the means to predict events individually, rather than having to wait for "patterns" to develop as is required with data-mining analyses. Further an *ATIS*-topological-analysis provides the means to effectively integrate and use metadata in a manner that provides real-time results and yet maintains personal security that is of concern to everyone in a free society.

Further, as a result of Yi Lin's work, it is now possible to analyze a system with multiple relations on a single component set. Such a capability makes *ATIS*-analyses possible for predicting results for intentional systems.

Also, as a result of Frick's work with respect to SIGGS, he defined the various affect-relations as being *basic*, *structural* and *dynamic*. From this, Thompson recognized that the structural relations defined the topology of an intentional system that gives rise to the ability to make predictions of an intentional system on a real-time basis. While the full value of topological analyses have not been developed in this report, the properties of a topology that should be of value are presented, as well as a constructive decision-procedure for determining a topology for any intentional system is given.

Also of value, an effective definition of *General System* is given as well as the *General System Object-Set*,  $G_0$ , *Construction Decision Procedure* and *Affect Relation-Set*,  $G_A$ , *Construction Decision Procedure* that clearly defines intentional systems so that they can be analyzed axiomatically. *Affect-Relation Properties* and *Information-Theoretic Properties* are also defined so that they can be incorporated into an *ATIS-Analysis*.

Further, the definition of *General System* is refined to provide more clarity (see page 18).

## Background Summary

In the 1920's, Ludwig von Bertalanffy envisioned a *General Systems Theory*<sup>1</sup>. As a biologist, von Bertalanffy was concerned with behavioral and intentional systems. He clearly stated the mathematical foundations of his theory in his report "The History and Status of General Systems Theory"<sup>2</sup>:

The goal obviously is to develop general systems theory in mathematical terms – a logico-mathematical field – because mathematics is the exact language permitting rigorous deductions and confirmation (or refusal) of theory.

In the 1960's, there were two major independent efforts made relating to developments in General Systems Theory. One was by the engineer and mathematician Mihajlo D. Mesarović, and the other was by the philosopher Elizabeth Steiner and the historian and mathematician George S. Maccia. The developments by Mesarović were more restrictive and in line with traditional developments of engineering models simulating various intentional systems, while the developments by Steiner and Maccia were more comprehensive and provided the first formalization of a *Scientific Education Theory* derived from *General Systems Theory*.

Mesarović's work, however, did lead to critical developments in mathematical models of General Systems; however, such characterizations were restricted to systems represented by a single relation.<sup>3</sup> A true mathematical analysis of *General Systems Theory* requires the ability to recognize multiple relations for one system. It would be another 30 years before that would be accomplished.

Also in the 1960's, Steiner and Maccia published their comprehensive treatment of *General Systems Theory* in developing a devising model for educational theory, the *SIGGS Theory Model*. This work was published in 1966, *Development of Educational Theory Derived from Three Educational Theory Models*.<sup>4</sup> This work was the first development of a **scientific or empirical education theory**. *A-GSBT* (subsequently changed to *ATIS*) is an extension of this work by Steiner and Maccia.

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<sup>1</sup> For Internet references, consider: <http://panarchy.org/vonbertalanffy/systems.1968.html>, [http://www.nwlink.com/~donclark/history\\_isd/bertalanffy.html](http://www.nwlink.com/~donclark/history_isd/bertalanffy.html), [http://en.wikipedia.org/wiki/Ludwig\\_von\\_Bertalanffy](http://en.wikipedia.org/wiki/Ludwig_von_Bertalanffy), <http://www.mind-development.eu/systems.html>, [http://www.isnature.org/Events/2009/Summer/r/Bertalanffy1950-GST\\_Outline\\_SELECT.pdf](http://www.isnature.org/Events/2009/Summer/r/Bertalanffy1950-GST_Outline_SELECT.pdf).

<sup>2</sup> Bertalanffy, Ludwig von (1972), "The History and Status of General Systems Theory", *Trends in General Systems Theory*, G.J. Klir (ed.).

<sup>3</sup> Mesarović, Mihajlo D. (1972), "A Mathematical Theory of General Systems," *Trends in General Systems Theory*, G.J. Klir (ed.).

<sup>4</sup> Maccia, Elizabeth Steiner, and George S. Maccia (1966), *Development of Educational Theory Derived from Three Educational Theory Models*, The Ohio State University Research Foundation, Columbus, Ohio.

In the 1980's, Theodore W. Frick extended the *SIGGS Theory Model* by classifying the SIGGS properties into three categories: [Basic](#), [Structural](#) and [Dynamic](#).<sup>5</sup>

The recognition of the SIGGS categories by Frick led Thompson, in the 1990's, to recognize that the *Structural Properties* define the [topology](#) of a system. Developed properly as a mathematical theory, SIGGS could now be developed in a manner that could utilize the power of mathematics in educational theorizing.

But, there was still one problem that had to be overcome in order to treat SIGGS or any [General Systems Theory](#) mathematically—how to treat multiple relations in a system mathematically? It was as a result of Mesarović's work that Yi Lin extended the mathematical model so that multiple relations could be considered with respect to a single system<sup>6</sup>.

This critical advancement by Lin in 1999 made it possible for Thompson to develop *ATIS* as an extension of SIGGS in a manner that the multiple relations of a system can be made mathematically precise. This advancement makes it possible to realistically recommend that *ATIS* can be used as a logical basis for intentional system models. In particular, the work of Frick has extended the *SIGGS Theory* in such a manner that his [SimEd](#) model for education can be founded on the *ATIS* theoretical base (an axiomatic logico-mathematical base), thus eliminating the need to rely on scenario-based models, as Mesarović and others have had to do.

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<sup>5</sup> Since this report is intended for both those who are very familiar with axiomatic theories and those who are not, in order to facilitate the understanding of those who are not, there will be numerous hyperlinks to other sources that define or discuss various terms used in this report.

<sup>6</sup> Lin, Yi (1999), *General Systems Theory: A Mathematical Approach*, Kluwer Academic/Plenum Publishers, NY.

## Critical Developments for a Logico-Mathematical Theory

In 1964, M.D. Mesarović, in “Foundations for a General Systems Theory,” recognized two distinct approaches to the representation of a system: The “*terminal approach*,” and the “*goal-seeking approach*.” The *terminal approach* is the conventional representation of system as an entity that looks at a system from the outside and defines it in terms of subset mappings, as is done in physics, chemistry, engineering, etc. While, as Mesarović notes, such systems could be defined as *goal-seeking systems*, such representation would be meaningless, artificial or trivial.

Due to the strong bias toward empirical theories designed from the *terminal approach*, and physics, in particular being the paradigm for empirical theory development, the development of intentional system theories based on a *goal-seeking approach* is much less understood, if recognized at all.

The *goal-seeking approach* incorporates an invariant base that defines the system’s goals. Further, the [affect relations](#) of the system are defined so that they are related to the attainment of the system’s goals. Such a system description results in the ability to predict the system’s behavior. That is, by defining an axiomatic description of a system, the means are then available to predict the system behavior—its end-target or predictive outcomes—under conditions that are different from its previous behaviors.

An axiomatic-based system description is critical for an intentional, behaviorally-predictive system. Predictions derived therefrom are not dependent on the result of previous behaviors, experiments or outcomes. **Predictions are dependent on a parametric analysis of an existing system state.**<sup>7</sup> A sequence of previous system states can define a dispositional system behavior, but are used, not as a definitive guideline for predicting future behavior, but as part of a comprehensive analysis of the existing system state.

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<sup>7</sup> A *parametric analysis* is an analysis of relationships between system components. A *nonparametric analysis* is an analysis of relationships between descriptive; that is, non-specific, and inferred relationships that a researcher may propose in the process of identifying system components in a [rough set](#). *Classical sets* contain elements (components) that are well-defined, and elements can be specifically determined as to whether or not they belong to the sets. *Fuzzy sets* contain elements (components) that are not well-defined or are vaguely defined so that it is indeterminate which elements (components) belong to the sets although other elements (components) may be well-defined as in *classical sets*. *Rough sets* are defined by topological approximations, which include elements (components) that are well-defined as in *classical sets*, and elements (components) that may or may not be in the set. These potentially *rough set* components are not *fuzzy set* elements (components) since they are not vaguely defined, they are just unknown concerning the set property.



Statistical analyses rely on past performance to predict future **group** behavior. *Statistical analyses can never be individually predictive.*

Axiomatic analyses rely on the internal structure of the system to determine its current goal-seeking behavior. Thompson emphasizes the critical nature of this observation—**predictions made with respect to intentional, behavioral systems are obtained as the result of the system structure at a given time.** The [structure](#) determines not only what is possible, but also the **intent** of the system as determined by its goal-seeking parameters.

It is recognized that the behavior of goal-seeking systems are much more complex than the behavior of terminal systems. However, systems can and do function in spite of their complexity. The problem, then, is to analyze the system in terms of its internal functioning structure, rather than by attempting to analyze each component of that structure. Components are considered in their relatedness to other components and how that relatedness helps to define the system structure. They are not considered in such minute detail that the structure; that is, the intent and behavior of the system is obscured.

While there are many disciplines pursuing the study of [General Systems Theory](#) (GST), none have gotten at the promise of providing a comprehensive intentional, behavioral theory envisioned by von Bertalanffy. These disciplines include [cybernetics](#), [dynamic systems theory](#), [control theory](#), [information theory](#), [set theory](#), [graph theory](#), [network theory](#), [game theory](#), [decision theory](#), [chaos theory](#), [complex adaptive systems theory](#), among others. Each has helped to answer questions within their defined areas of study, but none are behaviorally predictive.

C. Francois of the [International Society for the Systems Sciences](#) (ISSS) has addressed the unresolved problem of predictability within the behavioral sciences during a seminar on systemic inquiry and integration. He asserts that the reason the disciplines to date are not behaviorally predictive is that they fail to address one of the more important unresolved problems of GST—how to develop a system theory that describes multiple and shifting interrelations and interactions between numerous elements at various levels of complexity of a system.

To describe the complexity of a system cited by C. Francois, it is asserted that no piecemeal approach can lead to a good understanding of the structure and dynamics of the complex wholes. What ISSS claims is needed is a set of concepts and models that can be used to understand relationships and moreover, simultaneous, transient and shifting relationships. Their approach to the problem, however, is inadequate. Their approach is:

We must collect all synergetic concepts and models. We must integrate them in multiple cross ways. We should construct sets of any number of them and use these specific tools to resolve or at least better manage unresolved complex problems. [“Target Paper” by C. Francois, ISSS.]

**Such an approach by the ISSS is doomed to failure from the outset.** Existing concepts and models, due to their targeted specific objectives are inconsistent when combined. Further, integrating models that address specific subsystems do not thereby describe the entire system when combined—the whole is not simply an accumulation of its components, a basic tenant of *General Systems Theory* itself.

What must be developed is a comprehensive and consistent theory describing intentional (behavioral) systems. That is the focus of this research—to develop *ATIS* that is expressed by a rigorous definition of system, a comprehensive listing of axioms and a logico-mathematical derivation of its implications (theorems/hypotheses)—that is, its predictive results. This research will develop an axiomatic theory that uses the *Predicate Calculus*, *Mathematical Topology*, and *APT*<sup>8</sup> to analyze complex system relations.

Predictive results are possible due to the evaluations of the total interactions and connectedness of the different system components, rather than an analysis of each type of system relation individually. A further clarification is found by distinguishing *General Systems Theory* from *Cybernetics*. *Cybernetics* focuses on the **function** of a system; that is, how a system controls its actions via feedback mechanisms, how it communicates with other systems or with its own system components.

*General Systems Theory*, on the other hand, focuses on the **structure** of a system; that is, how a system changes as a result of structural modifications resulting from changing component relations, receiving input, emitting output, changing environmental relations, etc. Hence, the resulting predictability targeted by this research arises as a result of evaluating a system's structural changes in terms of known theoretical outcomes. Structural changes that result from specific system modifications are predictable by *Axiomatic Theory of Intentional Systems (ATIS)* in the same manner as physics predicts the behavior of the physical universe as founded on the appropriate theory of physics.

An additional concern of *Complexity Theory* must be addressed. “*Complexity Theory is the study of emergent order in what are otherwise very disorderly systems.*”<sup>9</sup>

In a sense, complex systems innovate by producing spontaneous, systemic bouts of novelty out of which new patterns of behavior emerge. Patterns, which enhance a system's ability to adapt successfully to its environment, are stabilized and repeated; those that do not are rejected in favor of radically new ones, almost as if a cosmic game of trial-and-error were being played.<sup>10</sup>

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<sup>8</sup> See Theodore W. Frick's reports at: <http://educology.indiana.edu/Frick/index.html>, and the reports listed under “Pattern Analysis”.

<sup>9</sup> McElroy, M.W. (2000), “Integrating Complexity Theory, Knowledge Management and Organizational Learning,” *Journal of Knowledge Management*, V.4, No.3, 2000, p. 196.

<sup>10</sup> *Ibid.*



Such a problem in *Complexity Theory* is what C.S. Peirce described as a tychistic event due to chance spontaneity within a system exhibiting synechistic (continuity) characteristics. The process of evolution is one such example of the tychistic-synechistic mechanism. However, with *ATIS* there is no mystery about such processes. Any tychistic event arises as system input, whether that is the result of genetic change or the intellectual contribution of an individual initiating a new social order.

There is no mystery when systems are properly analyzed. Air Force Colonel Warden 3<sup>rd</sup> recognizes the value of a system properly analyzed when he rightly asserts:

***“Terrorists are quite vulnerable when a proper analysis of a terrorist’s network system is made.”***<sup>11</sup>

The same is true of *Complex Systems* or *General Systems*.

## **A Purposeful Existence and Operation Implies Predictability**

A close examination of systems reveals that the interaction of system elements acts as if they were simple units that can be described by a set of a few variables. Their vast internal complexity is not directly manifested in their interactions.

“This property of behavioral systems is not accidental: If we were to allow the elements to reflect all their internal complexity in the interactions, then the system as a whole would most probably not be able to display any stable and predictable behavior.” A purposeful existence and operation implies predictability.<sup>12</sup>

Intentional systems are predictive by the very fact that they are *intentional*, and are the focus of this research. Further, that predictability is not out of reach when an analysis is made of the system structure; as opposed to a detailed analysis of system components from which an attempt is made to infer system behavior. *ATIS* does not provide a “causal analysis” for predictability. Past events provide a basis for determining the dispositional behavior of the system, but they do not predict future behavior. Behavior predictability is determined by system structure and not prior states. Prior states determine system dispositional behavior that defines the invariant initial system structure, but not causality nor predictability.

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<sup>11</sup> Warden III, Colonel John A. (1988), “The Enemy as a System,” *Air and Space Power Chronicles*, National Defense University Press Publication, 13 pages.

<sup>12</sup> Mikhailov, A. S. (1990), *Foundations of Synergetics, I: Distributed Active Systems*, Springer-Verlag, Berlin, p. 2.

A problem confronted by General System Theorists<sup>13</sup> is that of accounting for multiple types of relations in a system. As noted above, Y. Lin, in “A Model of General Systems” establishes that a *General Systems Theory* can be developed that defines more than one relation between the objects of the system.

Frequently a general system,  $(V,S)$ , is defined with respect to one type of relation as Mesarović has done:

$$S \subset \Pi\{V_i|i \in I\}.$$

Now, pursuant to Lin, assume that the set  $V$  has two relations defined by  $f$  and  $\gamma$ . Then, the system  $(V,\{f,\gamma\})$  is not a Mesarović system because the set  $\{f,\gamma\}$  cannot be written in a uniform relation symbol without changing the object set  $V$ .

In general, intentional systems will be of the form:  $(V, \cup_{i=1 \dots n} R_i)$ ; where  $i \neq j$  implies that  $R_i \neq R_j$ , and represents the number of different relations defined on  $V$ . These are the types of systems that concern theories to be developed from *ATIS*.

## Intentional Systems Theory

The *Steiner and Maccia Theory* (formerly, *Maccia and Maccia Theory*) of 1966 has led to the development of a true scientific behavioral (intentional systems) theory. Prior to this development, behavioral theories had been founded upon philosophical perspectives, a theoretical perspective from another science, the results of limited empirical research, hypotheses restricting the theory to a specific behavioral area, or an agenda, whether religious, political, or personal. Although they may purport to be scientific theories, they have not been well developed as scientific theories and none are comprehensive as a behavioral theory.

The theory model developed by Steiner and Maccia is the *SIGGS Theory Model*. *SIGGS* is an acronym for the theories that were used to develop the theory model. Those theories are: Set Theory, Information Theory, Graph Theory, and General Systems Theory.

From this theory model the educational theory is retroduced. To be retroduced means that content is added to the theory model to form the educational theory.<sup>14</sup>

<sup>13</sup> Such theorists as: [Ludwig von Bertalanffy](#), [Talcott Parsons](#), [Niklas Luhmann](#), [Béla Heinrich Bánáthy](#), [Howard Thomas Odum](#), [Eugene Pleasants Odum](#), [Peter Michael Senge](#), [Richard A. Swanson](#), and [Debora Hammond](#).

<sup>14</sup> Maccia, p. 117.

The purpose of the current research is to develop an *Axiomatic Theory of Intentional Systems*, *ATIS*, or as previously described, a *Behavioral Theory*. Such theory will be developed as a model that can be applied to a variety of intentional (behavioral) systems. In particular, it is intended that *ATIS* will be used as the logical basis for [SimEd](#). In particular, *ATIS* provides an *Options Set*, the [ATIS Option Set](#),<sup>15</sup> which can be used to develop an open-ended number of intentional system theories.

The intent of *SIGGS*, as stated in the *SIGGS Final Report* is:

“to set forth hypotheses [axioms] about human behavior and other factors involved in behavior irrespective of selected outcomes.”<sup>16</sup>

The *1966 Final Report* presented the hypotheses of the *Behavioral Theory*. While *SIGGS Theory* has been available since 1966, there has been little development of the theory since that time (with the exception of the work by [Frick](#) and [Thompson](#)), and it has received little attention as a prospective model for behavior theory development. The reason for this lack of attention has been recognized by Kira S. King and Theodore W. Frick in their article “Transforming Education: Case Studies in Systems Thinking.”<sup>17</sup> Therein they state:

Unfortunately, since *SIGGS* is written in highly complex mathematical language, it has received little attention since its creation.

A further reason is that *SIGGS* and *ATIS* are axiomatic theories, whereas current emphasis for practically all research is on statistical analyses; e.g., data mining technologies, and similar research.

The present work will do nothing to further resolve the problem of relying on a logico-mathematical theory. The present work is designed; in particular, to provide an extensive formalization of the theory, and to, in fact, extend the mathematical rigor of the theory. It will build on Steiner and Maccia’s 1966 work and the extension of that work by Frick. Further, Frick’s development of *APT* will be integrated into this extended theory as a tool for evaluating specific dynamic applications of the theory.

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<sup>15</sup> An implementation of the *ATIS Option Set* has five steps: (1) Identify the problem-statement that defines the components of the empirical system, (2) Identify the affect relations of the target system, (3) Analyze the affect relations to determine relevant properties, (4) The relevant properties identify the related axioms, and (5) From the related axioms, derive the theory-predicted outcomes, the theorems/hypotheses. (P. 10 on the referenced site.)

<sup>16</sup> Maccia, p. 118.

<sup>17</sup> King, Kira S. and Theodore W. Frick (2000), “Transforming Education: Case Studies in Systems Thinking,” *ed: Education at a Distance*, September, Vol. 14, No. 9.

In order to accomplish the integration of *APT* as a tool for *ATIS* analyses, *APT* as defined by Frick will be modified to read as follows:

*APT is a method for gathering information about observable phenomena of an individual system such that temporal patterns of events can be used as constants in [ATIS] to predict individual behavior and outcomes.*

Returning now to the [SIGGS Theory Model](#), hypotheses were developed from the **education** content given the theory model by the assigned properties. Frick subsequently classified those properties into **Basic**, **Dynamic** and **Structural Properties**. It is this classification that has led to the current research.

The properties defined by Frick are as follows:

<u><b>Basic Properties</b></u> are those properties that are descriptive of a system.
<u><b>Structural Properties</b></u> are those properties that show how system components are connected or related to each other.
<u><b>Dynamic Properties</b></u> are those properties that describe patterns in time as change occurs within or between a system and its negasystem.

Upon review of the work done by Frick, Thompson recognized that the **Structural Properties** represented the **behavioral topology**. It was recognized that such a topology would bring the power of mathematics to the behavioral sciences as it has to other scientific theories. Such power is needed if behavioral theory is to join the ranks of the other empirical sciences.

## What is Topology?

In its most general form, *topology* is concerned with how things are connected.

While it is frequently thought of in terms of geometrical forms, it is important to avoid this confusion.

*Geometry* is concerned with describing the shapes of things.

*Topology* is concerned with connectedness.

Thought of in this way, it helps to eliminate that confusion, and suggests applications not generally considered as being topological.

Stephen Barr, *Experiments in Topology*,<sup>18</sup> gives the following examples of topology applications:

It frequently happens that when getting a cup of coffee one forgets the cream. The trick, here, is not to go and get the cream, but to take the cup to it. The first way involves four trips: going for the cream, bringing it to the table, taking it back right away, and returning to the coffee. The other way involves two: taking the cup to the refrigerator and returning with the cup. This cannot be helpfully expressed geometrically, but the kind of sequential planning used, though arithmetical, belongs rather in topology. (p. 197)

That is, the problem is concerned with *connectedness*. And, topology is used frequently in everyday life:

Most descriptions of an objects location are topological, rather than geometrical: The coat is in your closet; the school is the fourth house beyond the intersection of this street and Route 32; The Pen of my Aunt is in the Garden.

Again, these problems are concerned with *connectedness*. The value of topology to behavioral theorizing is seen in the importance of the multitude of components in a behavioral system that are connected, and the importance of the kinds of connectedness.

## Topology and Behavioral (Intentional Systems) Theory

The value of theory in general, and behavioral theory in particular, is that theory provides a means of **predicting outcomes**. To date, behavioral sciences have had to rely on empirical testing to arrive at predictive assertions. That is, given a hypothesis, experiments must be conducted in order to *verify* the hypothesis.

The difficulty with all such testing and any conclusions derived therefrom is that they are dependent upon statistical measures that are only *group-predictive*, and not *individually-predictive*. A further and perhaps far more important difficulty is that when considering hypotheses, there is no assurance that different hypotheses actually have the same basic assumptions; and, in fact, they probably do not. Without the same basic assumptions for two different hypotheses, they cannot be incorporated into the same theory. This problem persists even for hypotheses that are designed to study the same or similar types of events. In fact, many times hypotheses are revisited in order to refute the findings of one as opposed to another by claiming that the very foundations of the hypotheses compromised the study.

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<sup>18</sup> Barr, Stephen, *Experiments in Topology*, Dover Publications, 1989.



However, by analyzing the *structure* of the behavioral (intentional) system, conclusions; that is, **predictions** can be obtained from a *parametric analysis* of the system (see footnote 7).

*Parametric analysis* is the analyzing of hypotheses of a theory based only upon its parameters.

An added value to this type of analysis is that predictions relating to intentional systems can be made from their *nonempirical structural parameters*.<sup>19</sup> In fact, this is the only feasible way to ever analyze an intentional system with any assurance of the reliability of any outcomes. The reason is due to the very large number of structures contained in even the smallest behavioral system. *ATIS* generates thousands of theorems which, when applied to specific intentional systems, will result in millions of possible *hypotheses* (that is, *theorems*) being generated.

Analyzing these systems by means of a *parametric analysis of their nonempirical structural parameters* appears to be the only reliable avenue to ever achieving the predictive results desired.

Further, it then becomes possible to evaluate a particular intentional system by first evaluating a formal system that is homeomorphic to the behavioral system.<sup>20</sup> Any topological invariants will be the same for both systems, thus eliminating the necessity of conducting empirical tests for each and every distinct behavioral system. If they are homeomorphic, predictions can be made from the formal system about the empirical system.

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<sup>19</sup> *Nonempirical Structural Parameters*, NeSPs, are discussed in a separate report—*QSARs, QSPRs, and their relevance to ATIS*. It is intended that this report will soon be published at some time in 2015.

<sup>20</sup> See [ATIS Properties: Morphisms](#).

## Primary Basic Properties

The [Basic Properties](#) define the attributes required for *General Systems Theory*. They are basic to the concept of a *General System*,  $\mathcal{G}$ . The first property, group, defines the (*component-*) *object-set*,  $\mathcal{G}_0$ , of a system.<sup>21</sup>

**Group**,  $\mathcal{G}_0$ , =<sub>df</sub> A set with at least two components within the universe of discourse.

$$\mathcal{G}_0 =_{df} \{ \mathbf{x} \mid \mathbf{x} \in \mathcal{W} \in \mathcal{U} \} \wedge |\mathcal{W}| > 1$$

In this definition, ‘ $\mathcal{U}$ ’ is the universe of discourse, ‘ $\mathcal{W}$ ’ is an object-set, and  $|\mathcal{W}|$  is the set-cardinality function.

As the initial intent of this research is to be able to analyze *complex intentional systems* with a multitude of elements, various types of elements, and numerous types of connectedness, an effective process must be established to identify those elements, the elements of  $\mathcal{G}_0$ . Further, as a General System will be analyzed as a topology, the topological relation-set will be introduced.

Although ‘General System’ has not yet been defined, ‘group’ is defined in anticipation of its future use as the General System Object Set,  $\mathcal{G}_0$ . Although, at this point, it is nothing more than a “group,” its construction is defined so as to be applicable to a *General System*.

In order to obtain precise property and affect relations’ definitions, the object-set must be precisely defined. The *General System Object-Set,  $\mathcal{G}_0$ , Construction Decision Procedure* is defined below.

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<sup>21</sup> In addition to the object-set, the relation-set and affect-relation-qualifiers are also *Basic Properties*. These will be defined as appropriate.

### General System Object-Set, $G_0$ , Construction Decision Procedure

The logical construction of the *General System Object-Set*,  $G_0$ , will be determined as follows:

- 1) Every *Information Base* ( $\bar{I}_B$ ) defines affect relations,  $\mathcal{A}_n \in \mathcal{A}$ , by the unary- and binary-component-derived sets from the  $\bar{I}_B$ . That is, the components of  $\mathcal{A}_n$  are of the form:  $\{\{\mathbf{x}_i\}, \{\mathbf{x}_i, \mathbf{y}_i\}\} \in \mathcal{A}_i \in \mathcal{A}_n$  that indicates that an “affect relation” has been empirically determined to exist from “ $\mathbf{x}_i$ ” to “ $\mathbf{y}_i$ .”
- 2) However, in order to even determine the affect relations, the qualifiers that specify the affect relations must be established. Very simply, what is the nature of the system being considered? To define the system, the affect relations must be known, and those are defined by the system qualifiers, the predicates that define the affect relations; and, therefore, the components of the system.

The *Affect Relation Qualifiers Set*,  $\mathcal{Q}$  must be defined before any affect relations, and, therefore, any components can even be recognized. These are the predicates that define which affect relations will be considered for system inclusion.

- 3) The following functions,  $\mu$  and  $\beta$ , define elements of a topology,  $\tau_n$ , that will allow for analysis of an affect relation. That is,  $\mu, \beta: \mathcal{A}_n \rightarrow \tau_n$ , such that:

$$\mu \mathcal{A}_i = \{\mathbf{x}_i\} \in \tau_n; \text{ and } \beta \mathcal{A}_i = \{\mathbf{x}_i, \mathbf{y}_i\} \in \tau_n.$$

An additional function,  $\varphi$ , will also be required for certain properties, and will allow for specification of specific elements, as follows:

$$\varphi \beta \mathcal{A}_i = \mathbf{y}_i.$$

Hence, the elements of  $G_0$  can be specified by  $\varphi$  and  $\mu \cap \beta$ .

- 4) The set of initial elements of  $G_0$  will be defined by an existing  $\bar{I}_B$  as follows:

$$G_0 = \{x \mid \exists i(x \in (\mu \mathcal{A}_i \cap \beta \mathcal{A}_i) \wedge \mathcal{A}_i \in \mathcal{A}_n)\}.$$

- 5) New elements will be added to  $G_0$  by Rule 3) when the new element establishes an affect-connected relation with an existing element in  $G_0$  so that it is an element of an  $\mathcal{A}_i \in \mathcal{A}_n$ .
- 6) No other objects will be considered as elements of  $G_0$  except as they are generated in accordance with Rules 1) to 4).

Now that the object-set has been determined, the concept of system must be established.

## System

There are various definitions of ‘system’ in the literature.<sup>22</sup> A Mesarović system is frequently used and it relates to the traditional concept of what a system “should” be; that is, it consists of related components. In this definition, a system is a relation on non-empty sets:

$$S \subset \prod\{V_i; i \in I\}; \text{ where ‘I’ is an index set.}$$

Lin extends the Mesarović definition so that multiple relations with a varying number of variables may be defined without having to change the object set, and defines a ‘system’, A, more conventionally as an ordered pair consisting of an object set, M, and a relation set, F:

$$A = (M, F).$$

Steiner and Maccia followed this convention and defined ‘system’ as follows:

**System,  $\mathfrak{S}$ ,** =<sub>df</sub> A group with at least one affect relation that has information.

$$\mathfrak{S} =_{df} (\mathbf{S}, \mathfrak{R}) = (\mathfrak{S}_0, \mathfrak{S}_\phi); \text{ where } \mathbf{S} = \mathfrak{S}_0 \text{ and } \mathfrak{R} = \mathfrak{S}_\phi.$$

A **system** is an ordered pair defined by an *object-set*,  $\mathbf{S}$  or  $\mathfrak{S}_0$ , and a *relation-set*,  $\mathfrak{R}$  or  $\mathfrak{S}_\phi$ .

In this research, the definition of system will be extended to more adequately account for all system parameters. This extension will more clearly define the topology and/or relatedness of a system by its object-sets and relation-sets; as well as allow for a more rigorous and comprehensive development of the system logic required for a logical analysis utilizing the Predicate Calculus and other required logics.

A *General System* is defined within a *Universe of Discourse*,  $\mathcal{U}$ , that includes the system and its environment. The only thing that demarcates the systems under consideration is the “*Universe of Discourse*.” And, while that universe may be somewhat *fuzzy* or *rough*, whatever systems are being considered will be well defined. In the case of *Education Systems*, the boundary of the universe may be quite fluid, or possibly unknown, especially with respect to the object-sets.

$\mathcal{U}$  is partitioned into two disjoint systems,  $\mathfrak{S}$  and  $\mathfrak{S}'$ . Therefore, *Universe of Discourse* has the following property:

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<sup>22</sup> See [System](#).

$$\mathcal{U} = \mathfrak{S} \cup \mathfrak{S}'; \text{ such that, } \mathfrak{S} \cap \mathfrak{S}' = \emptyset.$$

The disjoint systems of  $\mathcal{U}$ ,  $\mathfrak{S}$  and  $\mathfrak{S}'$ , are defined as “[system](#)” and “[negasystem](#),” respectively.

*System environment* and *negasystem environment* are defined as follows:

**System environment,  $\mathfrak{S}'$ ,** =<sub>df</sub> The system’s corresponding negasystem,  $\mathfrak{S}'$ .

**Negasystem environment,  $\mathfrak{S}$ ,** =<sub>df</sub> The negasystem’s corresponding system,  $\mathfrak{S}$ .

## General System

A *General System*,  $\mathcal{G}$ , is defined by the following parameters:

- |   |  |
|---|--|
| 1) <i>Family of Affect Relations Set, <math>\mathcal{A}</math>;</i> | 4) <i>Transition Function Set, <math>\mathcal{T}</math>;</i>       |
| 2) <i>Affect Relation Qualifiers Set, <math>\mathcal{Q}</math></i>  | 5) <i>Linearly Ordered Time Set, <math>\mathcal{T}</math>, and</i> |
| 3) <i>Component Partitioning Set, <math>\mathcal{P}</math>;</i>     | 6) <i>System State-Transition Function, <math>\sigma</math>.</i>   |

That is:

**General System** ( $\mathcal{G}$ ) =<sub>df</sub> a set of affect-relations ( $\mathcal{A}$ ) defined by affect-relation-qualifiers ( $\mathcal{Q}$ ), which determine a set of partitioned components ( $\mathcal{P}$ ), a transition functions set ( $\mathcal{T}$ ), a linearly-ordered time set ( $\mathcal{T}$ ), and a state-transition function ( $\sigma$ ). That is:

$$\mathcal{G} =_{df} (\mathcal{A}, \mathcal{Q}, \mathcal{P}, \mathcal{T}, \mathcal{T}, \sigma)$$

This definition is more accurately defined as follows:

$$\mathcal{G} =_{df} [\mathcal{A} | \mathcal{Q} \Vdash (\mathcal{P} (\mathcal{T}, \mathcal{T}, \sigma))];^{23}$$

That is, **General System**,  $\mathcal{G}$ , is defined as the *Affect-Relations Set*,  $\mathcal{A}$ , given the *Affect-Relation Qualifier Set*,  $\mathcal{Q}$ , which determine the *Component Partitioning Set*,  $\mathcal{P}$ , explicated by the *Transition Functions Set*,  $\mathcal{T}$ , the *Linearly-Ordered Time Set*,  $\mathcal{T}$ , and the *State-Transition Function*,  $\sigma$ .

---

<sup>23</sup> ‘ $\Vdash$ ’ is read “determines” or “which determine” or “from which is/are derived”, as appropriate for the sentence in which it is used. This symbol is similar in intent to the logical “yields”, but whereas “yields” is a logical relation for a deductive proof, this is a predicate relation identifying that which is derived from the existent set. This definition is used as it emphasizes the fact that no system can be recognized without first knowing the affect-relations as defined by the qualifying predicates. If no affect relation is recognizable, then no components can even be found.



The sets that define  $\mathcal{G}$  have the following elements:

$$\begin{aligned} &\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n \in \mathcal{A}; \text{ and} \\ &\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P, \mathcal{O}_P, \mathcal{S}_P, \mathfrak{S}_{B0}, \mathfrak{S}'_{B0} \in \mathcal{P}; \\ &\mathcal{L}, \mathcal{L}' \in \mathcal{Q}; \\ &\mathfrak{f}_I, \mathfrak{f}_O, \mathfrak{f}_T, \mathfrak{f}_B, \mathfrak{f}_S, \mathfrak{f}_N, \mathfrak{f}_E \in \mathcal{T}; \\ &t_1, t_2, \dots, t_k \in \mathcal{J}. \end{aligned}$$

Let the object-set of a General System,  $\mathcal{G}_0$ , be such that  $\mathcal{G}_0 = \mathfrak{S}_0 \cup \mathfrak{S}'_0$ ; where  $\mathfrak{S}_0$  and  $\mathfrak{S}'_0$  are the object-sets of  $\mathfrak{S}$  and  $\mathfrak{S}'$ , respectively. Then,  $\mathcal{G}_0$  is defined by the following:

$$\mathcal{G}_0 =_{\text{df}} \mathfrak{S}_0 \cup \mathfrak{S}'_0 = (\mathcal{I}_P \cup \mathcal{F}_P \cup \mathcal{S}_P \cup \mathfrak{S}_{B0}) \cup (\mathcal{T}_P \cup \mathcal{O}_P \cup \mathfrak{S}'_{B0})$$

Further, as all of these sets are disjoint, the following holds:

$$\mathcal{I}_P \cap \mathcal{F}_P \cap \mathcal{S}_P \cap \mathfrak{S}_{B0} \cap \mathcal{T}_P \cap \mathcal{O}_P \cap \mathfrak{S}'_{B0} = \emptyset.$$

$\mathcal{T}_P, \mathcal{I}_P, \mathcal{F}_P, \mathcal{O}_P, \mathcal{S}_P, \mathcal{L}, \mathcal{L}'$ ,  $\mathfrak{S}_{B0}$ , and  $\mathfrak{S}'_{B0}$  represent the following sets:

- ‘ $\mathcal{T}_P$ ’ represents “toput.”
- ‘ $\mathcal{I}_P$ ’ represents “input.”
- ‘ $\mathcal{F}_P$ ’ represents “fromput.”
- ‘ $\mathcal{O}_P$ ’ represents “output.”
- ‘ $\mathcal{S}_P$ ’ represents “storeput.”
- ‘ $\mathcal{L}$ ’ represents “system logisticians” or “system qualifiers.”
- ‘ $\mathcal{L}'$ ’ represents “negasystem logisticians” or “negasystem qualifiers.”
- ‘ $\mathfrak{S}_{B0}$ ’ represents “system background components.”
- ‘ $\mathfrak{S}'_{B0}$ ’ represents “negasystem background components.”

In view of the foregoing, the system object-set,  $\mathfrak{S}_0$ , and negasystem object-set,  $\mathfrak{S}'_0$ , are defined as follows:

$$\begin{aligned} \mathfrak{S}_0 &=_{\text{df}} \mathcal{I}_P \cup \mathcal{F}_P \cup \mathcal{S}_P \cup \mathcal{L} \cup \mathfrak{S}_{B0}; \text{ and} \\ \mathfrak{S}'_0 &=_{\text{df}} \mathcal{T}_P \cup \mathcal{O}_P \cup \mathcal{L}' \cup \mathfrak{S}'_{B0}. \end{aligned}$$

**Corollary:**

$$\mathfrak{S}_{B0} = \mathfrak{S}_0 \setminus (I_P \cup F_P \cup S_P); \text{ and } \mathfrak{S}'_{B0} = \mathfrak{S}'_0 \setminus (T_P \cup O_P).$$

*Background Components* may arise when the object-set is *fuzzy* or *rough* (see [fuzzy set theory](#) or [rough set theory](#)); that is, not all components are specifically known, but it is known that such components exist. For example, you may know that there are over 10,000 people in a particular town, but you do not know who they all are.

Now that the object-sets have been defined, the relation-sets must be defined.

*Transition functions* give the system dynamics. These are the functions that are operated on by the *System State-Transition Function*,  $\sigma$ , so as to change the system structure and thereby the “behavior” of the system.

*System behavior* is defined as a sequence of *system states*.

A consistent pattern of system states defines *System [Dispositional Behavior](#)*.

The transition functions required for state-transition analysis are described as follows:  $f_I, f_O, f_T, f_B, f_S, f_N, f_E$  are the transition function-sets and represent the following functions:

- |                             |                           |                             |
|-----------------------------|---------------------------|-----------------------------|
| ‘ $f_I$ ’ is “feedin.”      | ‘ $f_O$ ’ is “feedout.”   | ‘ $f_T$ ’ is “feedthrough.” |
| ‘ $f_B$ ’ is “feedback.”    | ‘ $f_S$ ’ is “feedstore.” | ‘ $f_N$ ’ is “feedintra.”   |
| ‘ $f_E$ ’ is “feedenviron.” |                           |                             |

## Affect Relations

Affect relations determine the structure of the system by the connectedness of the components.  $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_n$  are the affect relation-sets of  $\mathcal{G}$ . These sets are elements of the family of affect relations,  $\mathcal{A}$ . These sets define each subsystem of  $\mathcal{G}$ . For example, a *T/I-put interface system* will be defined as:  $T/I =_{df} T_P \cup I_P$ , and is defined by the affect relations that define the feedin function,  $f_I$ , that results in the input resulting from a *System State-Transition* of toput into the system,  $\mathfrak{S}$ . For example, this subsystem may have three affect relations,  $\mathcal{A}_1, \mathcal{A}_2$ , and  $\mathcal{A}_3$ , that will generate the transition functions,  $f_I$ . That is:

$$f_{I(1)} \subset \mathcal{A}_1, f_{I(2)} \subset \mathcal{A}_2, \text{ and } f_{I(3)} \subset \mathcal{A}_3.$$

Then, the *System State-Transition Function*,  $\sigma$ , operating on the transition functions,  $\mathcal{f}$ , “move” the qualified components from  $\mathfrak{S}'$  to  $\mathfrak{S}$  for each type of affect relation.

Steiner and Maccia define **affect relation** as follows:

**Affect relation,  $\mathcal{A}$** , =<sub>df</sub>

A connection of one or more components to one or more other components.

$$\mathcal{A} =_{df} \{ \{ \{ \mathbf{x} \}, \{ \mathbf{x}, \mathbf{y} \} \} \mid P(\mathbf{x}, \mathbf{y}) \mid \mathcal{Q} \wedge \mathbf{x}, \mathbf{y} \in \mathfrak{X} \subset \mathcal{G}_0 \vee [(\mathbf{x} = \mathcal{U} \subset \mathfrak{X} \subset \mathcal{G}_0 \wedge \mathbf{y} = \mathcal{V} \subset \mathfrak{Y} \subset \mathcal{G}_0)] \}$$

**Affect relations** define the connectedness of the system.<sup>24</sup>

In the current research, affect relation, as defined below, is a binary-relation of the form  $\{ \{ \mathbf{x} \}, \{ \mathbf{x}, \mathbf{y} \} \}$  as empirically derived from an  $\bar{I}_B$  (information base). If the direction of the affect relation is unknown, then both  $\{ \{ \mathbf{x} \}, \{ \mathbf{x}, \mathbf{y} \} \}$  and  $\{ \{ \mathbf{y} \}, \{ \mathbf{x}, \mathbf{y} \} \}$  will be included in the affect relation set.

This definition of affect relation is comparable to a Mesarović system, which is consistent with the current development since each relation defines a Mesarović system. Further, Mesarović refers to such systems as “input-output” systems, where

$$X = \prod \{ V_i \mid i \in I_X \}, \text{ the “inputs”}; Y = \prod \{ V_i \mid i \in I_Y \}, \text{ the “outputs”}; \text{ and}$$

$\{ I_X, I_Y \}$  is a partition of the index set,  $I$ . Since  $X \cap Y = \emptyset$ , the partition condition is satisfied. Now, this definition can be written to look very similar to that intended by Steiner and Maccia; that is:

$$\mathcal{A} \subset \mathfrak{X} \times \mathfrak{Y} = \{ (\mathbf{x}, \mathbf{y}) \mid \mathbf{x} \in \mathfrak{X} \wedge \mathbf{y} \in \mathfrak{Y} \}$$

And, from this, the family of affect relations can be obtained, such that:  $\forall n (\mathcal{A}_n \in \mathcal{A})$ .

As with the object-set, an effective procedure must be established for determining the elements of the affect relations. The *Affect Relation-Set,  $\mathcal{GA}$ , Construction Decision Procedure* is such an effective procedure and is given below.

<sup>24</sup> To be accurate, any predicate,  $P$ , should be defined as being derived from the affect-relation-qualifier set,  $\mathcal{Q}$ ; however, such will be assumed unless stated otherwise. That is,  $P(\mathbf{x}, \mathbf{y}) \mid \mathcal{Q}$  ( $P(\mathbf{x}, \mathbf{y})$  given  $\mathcal{Q}$ ) will simply be written as  $P(\mathbf{x}, \mathbf{y})$ .

**Affect Relation-Set,  $\mathcal{G}_{\mathcal{A}}$ , Construction Decision Procedure**

The logical construction of the affect relation-set,  $\mathcal{G}_{\mathcal{A}}$ , will be determined as follows:

1) *Affect Relation-Set Predicate Schemas*,  $P_n(\mathbf{x}_n, \mathbf{y}_n) | \mathcal{Q}_n = P_n(\mathcal{A}_n)$ , are defined as required to empirically define the family of affect-relations,  $\mathcal{A}_n \in \mathcal{A}$ , as extensions of the predicate schemas. The elements of  $\mathcal{A}_n$  are of the form  $\{\{\mathbf{x}\}, \{\mathbf{x}, \mathbf{y}\}\}$  that indicates that an “affect relation” has been empirically determined to exist from “ $\mathbf{x}$ ” to “ $\mathbf{y}$ .” ‘ $P_n(\mathcal{A}_n)$ ’ designates the predicate that defines the elements of  $\mathcal{A}_n$  as derived from the predicate-qualifier  $\mathcal{Q}_n$ .

2) The *Affect-Relation Transition Function*,  $\phi_n$ , is defined by:

$$\phi_n: \mathcal{X} \times \mathcal{Y} \rightarrow \mathcal{A}_n \mid \mathcal{X}, \mathcal{Y} \subset \bar{I}_B \ . \wedge \ . \phi_n(\mathcal{X} \times \mathcal{Y}) = \{ \{ \{ \mathbf{x}_n \}, \{ \mathbf{x}_n, \mathbf{y}_n \} \} \mid P_n(\mathcal{A}_n) \wedge \mathbf{x}_n \in \mathcal{X} \wedge \mathbf{y}_n \in \mathcal{Y} \} .$$

3) The family of affect relations,  $\mathcal{A} = \mathcal{G}_{\mathcal{A}}$ , is defined recursively by applications of the function in 2) for all elements in  $\bar{I}_B$  to each  $P_n(\mathcal{A}_n)$  defined in 1).

4) New components are evaluated for each  $P_n(\mathcal{A}_n)$  defined in 1) and included in the appropriate extension when the value is “true”.

5) No other objects will be considered as elements of  $\mathcal{A}_n \in \mathcal{A} = \mathcal{G}_{\mathcal{A}}$  except as they are generated in accordance with rules 1) through 4).

By convention,  $\{\{\mathbf{x}\}, \{\mathbf{x}, \mathbf{y}\}\} \equiv (\mathbf{x}, \mathbf{y})^{\rightarrow} \equiv (\mathbf{x}, \mathbf{y})$ , where the latter can be used if there is no confusion concerning direction of the relation.

## Transition Functions

The transition functions will now be defined in a manner to allow for temporal analysis of the system.

**Feed-Function Schema.** The “feed-” functions,  $f_F$ ; that is,  $f_I, f_O, f_T, f_B, f_S, f_N$ , and  $f_E$ , are defined as follows:

$$f_F: X_P \rightarrow Y_P \mid f_F(\mathbf{x}) = \mathbf{y}.$$

$X_P$  and  $Y_P$  are the corresponding “-put” sets defined for each function. For example,  $f_I: T_P \rightarrow I_P \mid f_I(\mathbf{x}) = \mathbf{y}$  defines the movement of toput components to input components.

**-Put Set Schema.** For all of the “-put” sets,  $P$ ; that is,  $T_P, I_P, F_P, O_P$ , and  $S_P$ , a **time function**,  $f(\mathbf{x})_{f_F(t)}$ , is defined from the product set of a “-put” set,  $f_F$ , and a time set into the real numbers,  $\mathcal{R}$ .

$$f(\mathbf{x})_{f_F(t)}: f_F \times \mathcal{T} \rightarrow \mathcal{R} = \mathcal{A}$$

For example,  $f_{T_P(t)}: T_P \times \mathcal{T} \rightarrow \mathcal{R} = \mathcal{A}$ ; that is,  $\mathcal{A}$  is the APT-value of  $T_P$  at time  $t$ .

To determine the temporal transitions of components, an APT-Analysis is performed with respect to the components of an affect relation such that:  $f_F(\mathbf{x}) = \mathcal{A}$ ; where  $\mathcal{A}$  is the APT value. When  $\mathcal{A}$  is greater-than or equal-to a predetermined value, or is 0, then component  $\mathbf{x}$  has “moved” to the target set as  $\mathbf{y}$ ; that is,  $f_F(\mathbf{x}) = \mathbf{y}$ . That is:

$$\exists \mathcal{A} [f_F(\mathbf{x} \in X_P) = \mathcal{A} \mid \mathcal{A} = 0 \vee \mathcal{A} > \alpha \supset f_F(\mathbf{x}) = \mathbf{y} \in Y_P]$$

**State-Transition Function Schema.** Then the **state-transition function**,  $\sigma$ , is defined by the following composition:

$$\sigma_x(f_F(\mathbf{x}) \circ f(\mathbf{x})_{f_F(t)}) = \mathbf{v} = 0 \supset \mathbf{x} \in f_F(\mathbf{x} \in X_P) = \mathbf{y} \in Y_P.$$



## Descriptive Analysis of General Systems

The descriptive analysis of an empirical system will be accomplished by using an [APT Analysis](#) developed by Frick. Further, the direct approach taken by an *APT Analysis* makes it readily applicable to a computer-based analysis of an  $\bar{I}_B$ . Frick describes the process as follows:

Analysis of patterns in time (APT) is a method for gathering information about observable phenomena such that probabilities of temporal patterns of events can be estimated empirically. [With an appropriate analysis] temporal patterns can be predicted from APT results.

The task of an observer who is creating an APT score [since renamed 'temporal map']<sup>25</sup> is to characterize simultaneously the state of each classification as events relevant to the classifications change over time.

An APT score ['temporal map'] is an observational record. In APT, a score ['temporal map'] is the temporal configuration of observed events characterized by categories in classifications.

[This contrasts significantly from the linear models approach (LMA) common to most research.] The worldview in the LMA is that we measure variables separately and then attempt to characterize their relationship with an appropriate mathematical model, where, in general, variable Y is some function of X. A mathematical equation is used to express the relation. In essence, the relation is modeled by a line surface, whether straight or curved, in  $n$ -dimensional space. When such linear relations exist among variables, then a mathematical equation with estimates of parameters is a very elegant and parsimonious way to express the relation.

In APT, the view of a relation is quite different. First, a relation occurs in time. A relation is viewed as a set of temporal patterns, not as a line surface in  $n$ -dimensional space. A relation is measured in APT by simply counting occurrences of relevant temporal patterns and aggregating the durations of the patterns. This may seem rather simplistic to those accustomed to the LMA, but Kendall (1973) notes,

“Before proceeding to the more advanced methods, however, we may recall that in some cases forecasting can be successfully carried out merely by watching the phenomena of interest approach. Nor should we despise these simple-minded methods in the behavioral sciences.”

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<sup>25</sup> While temporal information obtained from observing a particular system was initially referred to as an APT ‘score’ (e.g., Frick, 1990), the nomenclature was later changed to ‘temporal map’ as MAPSAT was further developed. See, for example: <https://www.indiana.edu/~tedfrick/MAPSATAECTORlando2008.pdf>, and <http://educology.indiana.edu/affectRelationTemporal.html>. This was necessary, since the most common meaning of ‘score’ is that of a number, such as the score in an athletic event or game. Frick was, however, using ‘score’ in the sense of a *musical* score that consisted of a temporal description of music by notation on staves for musicians to follow, e.g., to play *Beethoven’s 3rd Symphony*. Each musical ‘score’ is not a number; rather it is a configuration or map which is unique. Such a configuration is indexical and represents something unique; see for example: <http://educology.indiana.edu/sign.html>. Further, the musical map (score) for *Beethoven’s 3rd Symphony* is different from his 9th or his 5th symphonies. In addition, the distinction in MAPSAT between temporal maps and structural maps is consistent with the discussion of dynamic and structural properties of a given system.

For this research, *APT Analysis* lends itself quite readily to establishing patterns that indicate new objects and relations that should be added to the system. System state will be defined by system properties. System properties will be defined by the connectedness of the system components; which defines the system structure.

The most direct way to define the structure required is by utilizing graph-theoretic properties. These properties will be defined as required for the further development of *ATIS*.

## Affect Relation Properties, $\mathcal{A}$

*Affect Relation Properties* will be defined in terms of path-connected elements,  ${}_{pc}E$ . The properties are defined in set-theoretic terms so that they can be used to define a topology.

Therefore, before proceeding with the definitions of *Affect Relation Properties*, the relevant *Graph Theoretic Properties* will be presented.<sup>26</sup>

## Graph-Theoretic Connected Properties (Elements), ${}_{x}E$

**Path-connected elements,  ${}_{pc}E$ , =df**

$$\{(\mathbf{x}, \mathbf{y}) \mid (\mathbf{x} = \mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{n-1}, \mathbf{x}_n = \mathbf{y}) \wedge \forall (\mathbf{x}_i, \mathbf{y}_i)_{i < n} [\mathbf{y}_i = \mathbf{x}_{i+1}]\}$$

**Path-connectedness** is intuitively defined as the ability to get from one element to another by following a sequence of elements. The connected paths are “channels,” in terms of information theory, or “communications” between the elements of a system, or affect relations. These are graph-theoretic properties that will be used to define system properties.

**Discrete segment,  $|(\mathbf{x}, \mathbf{y})^{\rightarrow}_{n=1}| = 1$ , =df** A path between two and only two elements.

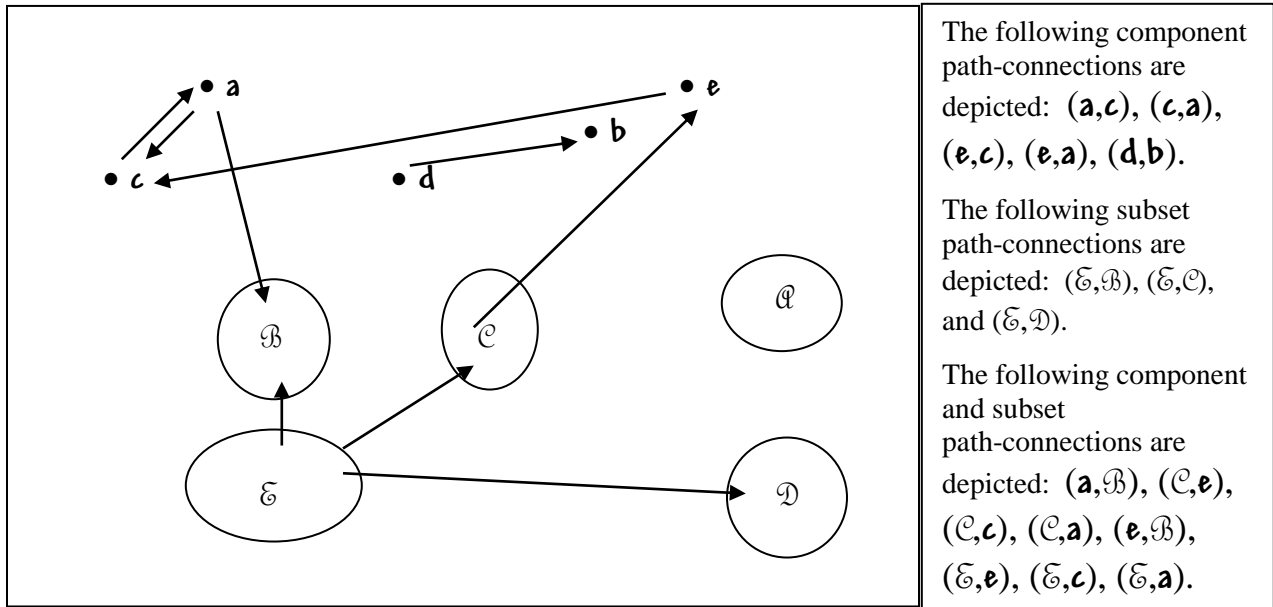
$$|(\mathbf{x}, \mathbf{y})^{\rightarrow}_{n=1}| = 1 \equiv \{(\mathbf{x}, \mathbf{y}) \mid (\mathbf{x} = \mathbf{x}_0, \mathbf{y} = \mathbf{x}_1)\}.$$

**Segment cardinality,  $|(\mathbf{x}, \mathbf{y})^{\rightarrow}_n| = n$ , =df** The number of discrete segments between elements.

$$|(\mathbf{x}, \mathbf{y})^{\rightarrow}_n| = n \equiv \{(\mathbf{x}, \mathbf{y}) \mid (\mathbf{x} = \mathbf{x}_0, \mathbf{y} = \mathbf{x}_n)\}.$$

The following graph depicts the path-connectedness of elements **a**, **b**, **c**, **d**, and **e**; and the path-connectedness of subsets,  $\mathcal{A}$ ,  $\mathcal{B}$ ,  $\mathcal{C}$ ,  $\mathcal{D}$ , and  $\mathcal{E}$ .

<sup>26</sup> For a more thorough discussion of graph theory for *ATIS*, go to [ATIS Graph Theory](#), and [ATIS: Connected Components and Affect Relations](#).



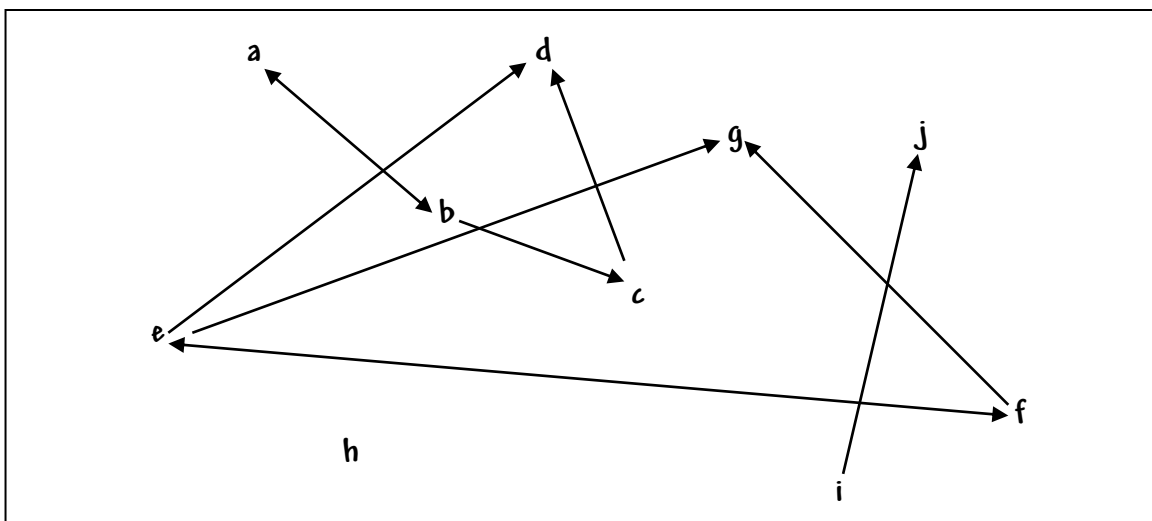
The following diagram and symbol conventions will be used to clarify and define the graph-theoretic properties.

Arrows ( $\rightarrow$ ,  $\leftrightarrow$ ,  $\leftarrow$ ) will be used to show direction of an affect relation between elements of a system.

' $(p,q)$ ' designates the **connected elements**  $p$  and  $q$ .

' $p \rightarrow q$ ' designates the ordered pair **path-connected elements** from  $p$  to  $q$ .

The following diagram, in addition to helping to clarify the connectedness properties, will also be used to introduce terminology that is useful for describing connectedness.



**Diagram of Directed Component Connectedness**

The following list is presented to facilitate the understanding of the various connectedness relationships. From the above graph, the following relations are determined:

**Path-connected elements:**

(a,b), (b,a), (a,c), (a,d), (b,c), (b,d), (c,d), (e,d), (e,f), (f,d), (f,e), (f,g), and (i,j).

**Path-connected elements with three segments:** (a,d).

**Completely connected elements:** (a,b) and (e,f).

**Unilaterally connected elements:** (a,c), (a,d), (b,c), (b,d), (e,d), (e,g), (f,g), and (i,j).

**Disconnected elements:** (a, h), (h, j), all h-pairs of elements, and all i and j pairs except for (i,j).

**Receiving elements:** a, b, c, d, e, f, g, and j.

**Initiating elements:** a, b, c, e, f, and i.

**Primary initiating elements:** i; that is, it initiates, but does not receive.

h may be considered as a trivial **primary initiating element**.

**Terminating elements:** d, g, and j.

h may be considered as a trivial **terminating element**.

All terminating elements must be **unilaterally terminating elements**.

**Connected but not path-connected elements:** (a,e), (a,f), (a,g), (b,e), (b,f), (b,g), (c,e), (c,f), and (c,g).

The terms described above will be formally defined below. **Path-connected elements** will be restated so as to bring all of the graph-theoretic properties together in one listing.

**Path-connected elements,  $_{pc}E$ ,** =<sub>df</sub>  $\{(x,y) \mid (x = x_0, x_1, x_2, \dots, x_{n-1}, x_n = y) \wedge \forall (x_i, y_i)_{i < n} [y_i = x_{i+1}]\}$

**Completely connected elements,  $_{cc}E$ ,** =<sub>df</sub>  $\{(x,y) \mid \forall (x,y)[(x,y), (y,x) \in _{pc}E]\}$

**Unilaterally connected elements,  $_{uc}E$ ,** =<sub>df</sub>  $\{(x,y) \mid \forall (x,y)[(x,y) \in _{pc}E \wedge (y,x) \notin _{pc}E]\}$

**Disconnected elements,  $_dE$ ,** =<sub>df</sub>  $\{x \mid \forall (x,y)[(x,y), (y,x) \notin _{pc}E]\}$

**Initiating elements,  $_iE$ ,** =<sub>df</sub>  $\{x \mid \forall x[(x,y) \in _{pc}E]\}$



**Receiving elements,  ${}_rE$ , =<sub>df</sub>  $\{\gamma \mid \forall \alpha[(\alpha, \gamma) \in {}_{pc}E]\}$**

**Terminating elements,  ${}_tE$ , =<sub>df</sub>  $\{\gamma \mid \forall \alpha[(\alpha, \gamma) \in {}_{pc}E \wedge \forall u(\gamma, u) \notin {}_{pc}E]\}$**

**Primary initiating elements,  ${}_{pi}E$ , =<sub>df</sub>  $\{\alpha \mid \exists \gamma[(\alpha, \gamma) \in {}_{pc}E \wedge \forall u(u, \alpha) \notin {}_{pc}E]\}$**

**Connected elements,  ${}_cE$ , =<sub>df</sub>  $\{(\alpha, \gamma) \mid \exists \gamma((\alpha, \gamma) \in {}_{pc}E \vee (\gamma, \alpha) \in {}_{pc}E)\}$**

The distinction must be made between **component properties** and **system properties**.

**Component properties** describe relations between components; for example, that two components are unilaterally connected.

**System properties** describe the characteristic pattern of all components of the system with respect to a specific component property; for example, the unilateral connections of the system components are such that the system is characterized by *strongness*.

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In view of the above Graph Theoretic developments, the *Affect Relation Properties* can now be defined. To bring all of the *Affect Relation Properties* together, ***affect relation*** will again be defined.

**Affect relation,  $\mathcal{A}$** , =df A connection of one or more components to one or more other components.

$$\mathcal{A} =_{df} \{ \{ \{ \mathbf{x} \}, \{ \mathbf{x}, \mathbf{y} \} \} \mid P(\mathbf{x}, \mathbf{y}) \wedge \mathbf{x}, \mathbf{y} \in \mathcal{X} \subset \mathcal{G}_0 \vee [(\mathbf{x} = \mathcal{U} \subset \mathcal{X} \subset \mathcal{G}_0 \wedge \mathbf{y} = \mathcal{V} \subset \mathcal{Y} \subset \mathcal{G}_0)] \}$$

**Directed affect relation,  ${}_d\mathcal{A}$** , =df An *affect relation* that is path-connected.

$${}_d\mathcal{A} =_{df} \mathcal{A} \mid (\mathbf{x}, \mathbf{y})^{\rightarrow} \in \mathcal{A} \supset (\mathbf{x}, \mathbf{y})^{\rightarrow} \in {}_{pc}E \}$$

**Directed affect relations** may pass through more than one component. **Directed affect relations**, when also assigned a “magnitude” will be interpreted as a vector that will allow for topological analyses of the system vector fields.

**Direct directed affect relation,  ${}_{dd}\mathcal{A}$** , =df A *directed affect relation* with a single directed-path.

$${}_{dd}\mathcal{A} =_{df} \{ (\mathbf{x}, \mathbf{y})^{\rightarrow} \mid (\mathbf{x}, \mathbf{y})^{\rightarrow}_{n=1} \in \mathcal{A}_m \in \mathcal{A} \}$$

**Indirect directed affect relation,  ${}_{id}\mathcal{A}$** , =df

A *directed affect relation* in which the path-connection is through other components.

$${}_{id}\mathcal{A} =_{df} \{ (\mathbf{x}, \mathbf{y})^{\rightarrow} \mid (\mathbf{x}, \mathbf{y})^{\rightarrow}_{n>1} \in \mathcal{A}_m \in \mathcal{A} \}$$

**Connected affect relation,  ${}_c\mathcal{A}$** , =df

Connected components of an *affect relation* irrespective of direction of path-connectedness.

$${}_c\mathcal{A} =_{df} \{ (\mathbf{x}, \mathbf{y}) \mid (\mathbf{x}, \mathbf{y}) \in \mathcal{A}_m \in \mathcal{A} \vee (\mathbf{y}, \mathbf{x}) \in \mathcal{A}_m \in \mathcal{A} \}$$

Connected affect relations may be used to analyze a system in terms of its total connectedness to determine potential behaviors under varying assumptions of connectedness.

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## Information-Theoretic Properties

**Information** obtained from an *Information Base*,  $\bar{I}_B$ , will be analyzed to determine various affect relations. An *APT Analysis* will provide a sequence of system states that may be used to define various *Dispositional Behaviors*,  $\mathfrak{B}$ 's. Further, the  $\bar{I}_B$  will be used to construct an *Extended- $\bar{I}_B$*  that will be used to make predictions concerning system behavior. The *Extended- $\bar{I}_B$*  is constructed using the *Behavior-Predictive Algorithm* (the *Phoenix Algorithm*) developed by *Raven58 Technologies*.

**Information** is made explicit for analysis by the use of *mathematical probabilities*. *Probabilities* define **information**. And, the probabilities used do not have to be “true”; they only have to lend themselves to a proper analysis of the system and its outcomes—its predictions.

In *ATIS*, the probabilistic definition of **information** is mitigated by the fact that behavioral predictions are **not** founded on the information, but on a structural analysis of the system derived from that information. That is, *behavior prediction* made possible by *ATIS* is dependent on *logical and topological analyses* rather than on the specific **information** input itself. **Information** for *ATIS* is used to determine *system structure* and is **not** the decision-making tool.

Further, information as used in *ATIS* is discrete. As the information “H” function is defined below, *ATIS* only uses a few discrete values of “H,” normally equal to “0” or “not 0.”

For example, input occurs when the value of “H” in the toput is such that  $H = 0$ . If “H” is anything other than 0, then the component is still toput, regardless of whether  $H = 0.1$ ,  $H = 0.2$ ,  $H = 0.7$ , etc. However, various analyses of H will be used to construct the *Extended- $\bar{I}_B$* . That is, the value of H will determine the category assigned a new system component so that the new system structure may be determined and analyzed.

**Information** is that which reduces **uncertainty**. In information theory, **uncertainty** is defined by a value, H, the entropy. **Uncertainty** is a *measure* of **variety** such that **uncertainty**, H, is zero when all elements are in the same **category**.

Information is defined as follows:

**Information,  $p$ ,** =<sub>df</sub> A mathematical probability of occurrences defined by:

$$p =_{df} \{(c,v) | c \in W \subset G_0 \wedge v \in (0,1]\}.$$

**Information** is a set of *ordered pairs* consisting of components,  $c$ , of the set “ $W$ ”, a subset of  $G_0$ , and the real number “ $v$ ,” which is the probability distribution,  $p$ , that the *component* “ $c$ ” occurs in  $W$ .

**Information** is represented as a probability so as to convey the *uncertainty* of that **information**. Thus, **information** will be the result of an “*uncertain event*,” and referred to below as “*event uncertainty*.” **Information** is a *Measure Property*.

**Event uncertainty,  $H$ ,** =<sub>df</sub> A measure of information due to statistical uncertainty, real uncertainty, or enemy action.

$$H = -K \sum_{i=1 \dots n} p_i \log p_i;$$

where “ $K$ ” is a constant related to the choice of a unit of measure, and “ $p_i$ ” is the probability of occurrence of event “ $i$ ”. In *ATIS*, the **information-probability** will indicate the “reliability” of that information and the resulting assignment to an appropriate subset.

‘**Event uncertainty**’ is used here to imply probabilities that are subject to objective determination. Further, whereas ‘**event uncertainty**’ may be defined in terms of *statistical uncertainty* or *real uncertainty*, the *information* in this research will normally be due to *enemy action*; that is, tychistic events that must be dealt with in the continuity of a behavioral system, the society.

**Non-conditional event uncertainty,  $_{nc}H$ ,** =<sub>df</sub>

*Information* that does not depend on other event uncertainty.

$$_{nc}H =_{df} fH \mid \sim \exists fH_1 (fH: fH_1)$$

**Conditional event uncertainty,  $_{c}H$ ,** =<sub>df</sub> *Information* that depends on other event uncertainty.

$$_{c}H =_{df} fH: \exists fH_1 (fH: fH_1)$$

## MATHEMATICAL TOPOLOGY & BEHAVIORAL (Intentional Systems) THEORY

In this study, we will not be considering the study of topology, but will be considering the application of topology to behavioral theory. The applicability of topology to behavioral theory is suggested by [Klaus Jänich](#) in his text *Topology*<sup>27</sup> at page 2:

*“... the application of point-set topology to everyday uses in other fields is based not so much on deep theorems as on the unifying and simplifying power of its system of notions and of its felicitous terminology.”*

It is this “unifying and simplifying power” that we will develop for intentional systems theory. Following this development, the full power of mathematical topology in terms of its analytic tools and theorems will depend upon the intentional systems involved and the creativity of the analyst. Specific applications will be suggested throughout this development, and a constructive approach to analyzing behavioral topological spaces will be presented. Various definitions and theorems will be presented just to indicate what topological analyses may be of value in analyzing specific intentional systems.

### Topologies and Topological Spaces

There are a number of ways that a [topological space](#) can be defined. The one chosen herein was selected due to its potential for properly evaluating the desired concepts of the intentional systems theory.

[Topology](#) is the study of those properties of a system that endure when the system is subjected to topological transformations. This introduces the initial rationale for interpreting the structural properties as a topology—to be able to analyze properties of a system in a manner that can distinguish between substantive and non-substantive distinctions. Having initiated this interpretation, however, it may be seen that other interesting and beneficial evaluations will arise.

A **topological transformation** is a continuous transformation that has a continuous inverse transformation. That is, a **topological transformation** is a continuous transformation that can be continuously reversed or undone. Two systems are **topologically equivalent** if there is a topological transformation between them.

Thus, the tools are available to determine whether or not two intentional systems are in fact substantively different, or if they only appear to be different due to our vision being determined by geometric perspectives. That is, things are normally differentiated “geometrically,” whereas differentiating them topologically will get at their substantive distinctions. That is, normally distinctions may be “seen” where in fact there are none.

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<sup>27</sup> Jänich, Klaus, *Topology*, Springer, 1995.

A **topological property**; i.e., **topological invariant**, of a system is a property possessed alike by the system and all its topological equivalents. A **topological invariant** always carries information concerning one or more topological properties.

**Topological space** and **topology** are defined as follows.

**Definition:** *Topological Space and Topology.*

$T = (\mathfrak{S}_x, \tau)$  is a **topological space**, where  $\mathfrak{S}_x$  is a **set of points** and  $\tau$ , the **topology**, is a class of subsets of  $\mathfrak{S}_x$ , called **open-neighborhoods**, such that:

- (1) Every point of  $\mathfrak{S}_x$  is in some open-neighborhood,  $\mathcal{N}_i$ ,  $i \in \mathbb{I}^+$ , the set of positive integers;
- (2) The intersection of any two open-neighborhoods of a point contains an open-neighborhood of that point; and
- (3)  $\mathfrak{S}_x$  and  $\emptyset$  are elements of  $\tau$ .

Formally:

$$\tau = \{ \mathcal{N}_i \mid \mathcal{N}_i \subseteq \mathfrak{S}_x \wedge i \in \mathbb{I}^+ \wedge \forall x \in \mathfrak{S}_x \exists \mathcal{N}_i (x \in \mathcal{N}_i \wedge (x \in \mathcal{N}_j \wedge x \in \mathcal{N}_k \supset \exists \mathcal{N}_i (x \in \mathcal{N}_i \subset \mathcal{N}_j \cap \mathcal{N}_k)) \cup \{ \mathfrak{S}_x, \emptyset \}.$$

That is,  $\tau$  is the **topology** that consists of **open-neighborhood sets**,  $\mathcal{N}_i$ , of  $\mathfrak{S}_x$  such that every point of  $\mathfrak{S}_x$  is in some  $\mathcal{N}_i$ , and the intersection of any two neighborhoods of a point of  $\mathfrak{S}_x$  contains a neighborhood of that point, and  $\mathfrak{S}_x$  and  $\emptyset$  are elements of  $\tau$ .

## Properties of Topological Spaces for an Intentional Systems Theory

It is proposed that the following properties of topological spaces will be useful in analyzing intentional systems. Where appropriate, especially for vector fields, these properties are defined specifically for the intentional systems theory.

**Definition.** *Open Sets.*

$T$  is a topological space,  $(\mathfrak{S}_x, \tau)$ . The elements of  $\tau$  are **open sets**.

**Definition.** *Neighborhood of a Point.*

The set,  $\mathcal{N}$ , is a neighborhood of a point,  $p$ , if: (1)  $p \in \mathcal{N}$ ; and (2)  $\mathcal{N}$  is open.

**Definition.** *Near (Point-to-Set).*

$T$  is a topological space,  $(\mathfrak{S}_x, \tau)$ . Let  $S \subset \mathfrak{S}_x$  and  $x \in S$ .  $x$  is **near S**,  $x \leftarrow S$ , if every neighborhood of  $x$  contains an element of  $S$ .

**Definition.** Let  $T = (\mathfrak{S}_x, \tau)$ . Let  $S \subset \mathfrak{S}_x$ . Then:

Path.

Any set topologically equivalent to the line segment  $[0,1]$ .

Closed Path or Jordan Curve.

Any set topologically equivalent to the circle,  $[P] = 1$ .

Closed.

$S$  is **closed** if it contains all its near points.

That is, there are no neighborhoods other than  $S$  that contain the elements of  $S$ .

Open.

$S$  is **open** if every element in  $S$  is not near the complement of  $S$ ,  $S'$ .

That is, there are no neighborhoods for the elements of  $A$  which contain elements of  $S'$ .

Clopen.

$S$  is **clopen** if it is both open and closed.  $[\mathfrak{S}_x$  and  $\emptyset$  are always clopen in a topology.]

Connected.

$S$  is **connected** if for every nonempty disjoint partition of  $S$ ,  $U$  and  $V$ , one partition contains an element near the other.

**Theorem:** *Paths are connected.*

Following are additional topological properties that will be of value in analyzing various intentional systems.



**Definitions:** Let  $T = (\mathfrak{S}_x, \tau)$ . Let  $S \subset \mathfrak{S}_x$ . Then:

*Interior Point.*

$x \in \mathfrak{S}_x$  is an **interior point** of  $S$  if  $S$  is a neighborhood of  $x$ .

*Exterior Point.*

$x \in \mathfrak{S}_x$  is an **exterior point** of  $S$  if  $S'$  is a neighborhood of  $x$ .

*Boundary Point.*

$x \in \mathfrak{S}_x$  is a **boundary point** of  $S$  if neither  $S$  nor  $S'$  is a neighborhood of  $x$ .

*Set Interior.*

If  $S^\circ$  is the set of interior points of  $S$ , then  $S^\circ$  is the **interior** of  $S$ .

*Set Closure.*

If  $S^-$  is the set of points of  $S$  which are not exterior points, then  $S^-$  is the **closure** of  $S$ .

That is,  $S^-$  is the set of interior and boundary points of  $S$ .

**Definition:** *Disjoint Union (Sum) of Sets.*

Let  $X + Y = X \times \{0\} \cup Y \times \{1\}$ , then  $X + Y$  is the **disjoint union** or **sum** of  $X$  and  $Y$ .

By this definition we obtain a copy of  $X$  and  $Y$  individually, rather than their union.

With the following definition, we can obtain a new topology from two given topologies as follows.

**Definition:** *Disjoint Union (Sum) of Topological Spaces.*

If  $(X, \tau)$  and  $(Y, \tau^*)$  are topological spaces, then a new topology on  $X + Y, \tau^{**}$ , is given by:

$$\tau^{**} = \{U + V \mid U \in \tau \text{ and } V \in \tau^*\},$$

and  $(X + Y, \tau^{**})$  is called the topological disjoint union of the topological spaces  $X$  and  $Y$ .

This may prove fruitful when considering different topologies within the same behavioral system, or when trying to join two different systems.

We will now consider the definition of a **product topology**. The generalization of this definition will require the following definition of **product set**.

**Definition:** *Product Set.*

Let  $\{X_i | i \in I^+\}$  be a family of sets.

[**NOTE:**  $I^+$  may be replaced with a finite index set, and will generally be done so with a particular intentional system.]

Then,

$$X_1 + X_i = \{(x_1, x_2, \dots, x_i, \dots) | \forall i \in I^+ (x_i \in X_i)\}$$

If  $x = (x_1, x_2, \dots, x_i, \dots) \in X_1 + X_i$ , then,

$x_i$  is called the **i-th coordinate** of  $x$ ;

$X_i$  is called the **i-th component** of  $X_1 + X_i$ ; and

$X_1 + X_i$  is called the **product set** of the sets  $\{X_i | i \in I^+\}$

**Definition:** *Product Topology.*

If  $(X, \tau)$  and  $(Y, \tau^*)$  are topological spaces, then  $\tau^{**}$  is the **product topology** of  $X \times Y$  and is given by:

$$\tau^{**} = \{(x, y) | \exists \mathcal{N}(x) \exists \mathcal{N}(y) (x \in X \wedge y \in Y \supset \mathcal{N}(x) \in \tau \wedge \mathcal{N}(y) \in \tau^*)\}.$$

Generalizing this definition, we have the following:

Let  $\{(X_i, \tau_i) | i \in I^+\}$  be a countable family of topological spaces. Then,

$$\tau = \{X_1 + X_i | i \in I^+ \wedge \forall x_i \in X_i \exists \mathcal{N}(x_i) (x_i \in X_i \Rightarrow \mathcal{N}(x_i) \in \tau_i)\}$$

is the **product topology** of  $X_1 + X_i$ .

**Definition:** *Continuous.*

$S$  and  $T$  are topological spaces. A transformation,  $f:S \rightarrow T$ , is **continuous** if for any point,  $x \in S$ , and subset,  $A \subset S$ ,  $x \leftarrow A \Rightarrow f(x) \leftarrow f(A)$ .

**Definition:** *Bijjective Map.*

$f:X \rightarrow Y$ , is a **bijjective map** if  $f$  is 1-1 and onto.

**Definition:** *Homeomorphism.*

$f:X \rightarrow Y$ , is a **homeomorphism** if  $f$  is bijective and continuous, and  $f^{-1}$  is continuous.

The following connectedness theorems may prove useful when analyzing various intentional systems.

**Theorem:** *Continuous Images Maintain Connectedness.*

If  $X$  is a (path-)connected space and  $f:X \rightarrow Y$  is continuous, then  $f(X)$  of  $Y$  is also a (path-)connected space.

**Theorem:** *Non-Disjoint Unions Maintain Connectedness.*

If  $X_0$  and  $X_1$  are (path-)connected subspaces of  $X$ ,  $X = X_0 \cup X_1$ , and  $X_0 \cap X_1 \neq \emptyset$ , then  $X$  is (path-)connected.

**Theorem:** *Products Maintain Connectedness.*

$X_1 \times X_i$  of non-empty topological spaces is (path-)connected if and only if all  $X_i$  are.

## Topological Vector Fields

For the following definitions:  $T = (\mathcal{S}, \tau)$  is a topological space and  $P, Q \subset \mathcal{S}$ .

**Definition:** *Direct Affect Relation.*

$\mathbf{A}$  is a **direct affect relation** from  $x$  to  $y$  defined by  $\mathbf{A}: P \rightarrow Q$ , where  $x \in P$  and  $y \in Q$ , such that  $\{x, y\} \in \tau$ , and  $\mathbf{A}(x) = y$  is defined for all  $x$  of  $P$ .

**Definition:** *Direct Affect Relation Measure.*

The function,  $M$ , defined by  $M: P \times Q \rightarrow \mathcal{R}$ , is a **direct affect relation measure** of the direct affect relation,  $\mathbf{A}$ , defined by  $\mathbf{A}(x) = y$ , such that  $M(x, y) = m$ .

**Definition:** *Vector.*

$\mathbf{v}$  is a **vector** from  $x$  to  $y$ ,  $x \rightarrow y$ , if there is a direct affect relation,  $\mathbf{A}$ , and a direct affect relation measure,  $M$ , defined for  $x \in P$  and  $y \in Q$ .

**Definition:** *Vector Field.*

$V$  is a **vector field**, if  $V = \{(x, y) | x \in P, y \in Q, x \rightarrow y\}$ .

The value of *APT* methodologies now becomes quite apparent. The intentional systems theory provides the parametric formulas for analyzing an intentional system. *APT* provides the means for obtaining the information required to apply these analyses to specific intentional systems.

In particular, *APT* makes it possible to evaluate the family of **Affect Relation Vector Fields**. That is, any behavioral system will have numerous vector fields, which normally would have to be analyzed individually. *APT* provides the means to evaluate them simultaneously. That is, they are viewed in terms of the *APT Map*. This should prove to be most beneficial when applying the theory to a particular individual or system. *APT Maps* provide the methodologies to make the theory individually predictive or system-specific predictive.

Further, the theory, in terms of either the disjoint union or product of topological spaces will provide the theoretical perspective required to assign appropriate parameters to the *APT Map*, and then to analyze the results of that score in terms of their theoretical significance.

**Definition:** *Fixed Point and Fixed Point Property.*

Let  $f$  be a continuous transformation from  $X$  into  $X$ , represented by:

$$f_1: X \rightarrow X, f_2: X \rightarrow X, f_3: X \rightarrow X, \dots, f_i: X \rightarrow X, \dots, f_n: X \rightarrow X.$$

Then, if there is an  $x \in X$  such that  $f_1(x) = f_n(x) = x$ , then  $x$  is called a **fixed point** of  $f$ . If for every  $f$ ,  $X$  has a fixed point, then  $X$  has the **fixed-point property**.

**Theorem:** *Brouwer's Fixed Point Theorem.*

Cells have the fixed-point property.

**Definition:** *Self-Similar.*

A system that is homeomorphic to any subset of the system.

## Constructive Development of a Topology for an Intentional System

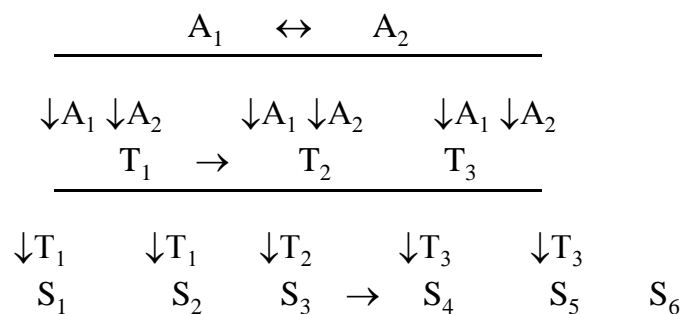
In order to consistently analyze intentional systems, a “Construction Rule” will be developed that will generate a topology in the same manner for every system analyzed.

### *Topology Construction Rule:*

- (1) Every element of the system is contained in a neighborhood consisting of one element, and are designated as the family of null-affect neighborhoods,  $A_0$ .
- (2) Every element of the system is classified by type and is assigned to the neighborhood containing just elements so classified. All such neighborhoods are designated as the family of descriptive neighborhoods,  $D_0$ .
- (3) All elements with affect relations between them are pair-wise assigned to a neighborhood containing just those two elements, and are designated as a family of affect relations by type; that is,  $A_1, A_2, A_3$ , etc.
- (4) The system and the null set are elements of the topology.

Now, let’s see how this works.

Let  $S = \{A_1, A_2, T_1, T_2, T_3, S_1, S_2, S_3, S_4, S_5, S_6\}$ ; where “A” represents “Armed Forces Personnel,” “T” represents “Terrorist Groups,” and “S” represents “Sites Targeted.” Graphically define the following affect relations.



By the above *Topology Construction Rule*, this generates the following neighborhoods for the topology:

$$A_0 = \{\{A_1\}, \{A_2\}, \{T_1\}, \{T_2\}, \{T_3\}, \{S_1\}, \{S_2\}, \{S_3\}, \{S_4\}, \{S_5\}, \{S_6\}\}$$

$$D_0 = \{\{A_1, A_2\}, \{T_1, T_2, T_3\}, \{S_1, S_2, S_3, S_4, S_5, S_6\}\}$$

$$A_1 = \{\{A_1, A_2\}, \{A_1, T_1\}, \{A_1, T_2\}, \{A_1, T_3\}, \{A_2, T_1\}, \{A_2, T_2\}, \{A_2, T_3\}\}$$

$$A_2 = \{\{T_1, T_2\}\}; \quad A_3 = \{\{T_1, S_1\}, \{T_1, S_2\}, \{T_2, S_3\}, \{T_3, S_4\}, \{T_3, S_5\}\}; \quad A_4 = \{\{S_3, S_4\}\}$$

Thus,

$$\tau = A_0 \cup D_0 \cup A_1 \cup A_2 \cup A_3 \cup A_4 \cup \{S, \emptyset\}.$$

This topology can be easily verified. And, the construction can be easily determined to always result in a topology, since every intersection will result in either a set of one element, which is in the topology, or in a set of two elements that is in the topology.

Now, let's see what may be determined that may not otherwise be too obvious. Granted, given this limited intentional system, this result may be somewhat obvious, but it does indicate the power of this analysis given much larger systems.

Are  $(T_1, S_4)$  and  $(T_1, S_5)$  path-connected?

The following sequence of neighborhoods determines that both pairs are path-connected.

For  $(T_1, S_4)$ :  $\{T_1, T_2\}, \{T_2, S_3\}, \{S_3, S_4\}$ .

For  $(T_1, S_5)$ :  $\{A_1, T_1\}, \{A_1, A_2\}, \{A_2, T_3\}, \{T_3, S_5\}$ .

While, intuitively, it may seem that  $(T_1, S_5)$  are not path-connected due to the direction of the arrows; that is, the affect relations, an analysis of the system indicates that they are so connected. This should not be too surprising, since the impact of the "system" concept is that elements of the system are in fact responsive to changes in or influences by other elements of the system. This does not mean that "vectored direct affect relations" cannot be determined; however, the question here is, do affects by  $T_1$  influence, or have an effect on,  $S_5$ ? The answer is "Yes." That degree of influence is not yet determined.



Now, another result of the above example is to see that an element may be connected but not path-connected. For example,  $S_6$  is connected but not path-connected. As seen from the example, there are no affect relations connecting  $S_6$  to the other elements. In the topology, there are only two neighborhoods that contain this element; that is,  $\{S_1, S_2, S_3, S_4, S_5, S_6\}$  and  $\{S_6\}$ . They, clearly, are not path-connected. But, are they connected? Yes. They are connected by the subset  $\{\{S_1, S_2, S_3, S_4, S_5, S_6\}, \{S_6\}\}$ . That is, when divided into the only two non-empty disjoint parts, then one part contains a point near the other, specifically  $S_6$ .

This is certainly a desired result in an intentional system. Since, obviously, a new Site Target can be introduced into the system at any time. Yet, at the time of introduction, there may not be any substantive affect relation established. The topological space must allow for that relation to be established. Of course, at the time of introduction into the system, certain affect relations may be established, but, not necessarily those under consideration. Clearly, a behavioral topology can become extremely complex. That complexity is being minimized for the sake of introducing topology as a tool for eventual analysis of that system's complexity.

Now for a consideration of the *vectorized direct affect relations*.

The following concepts are derived from Michael Henle's *A Combinatorial Introduction to Topology*.

A vector field  $V$  on a subset  $D$  of the plane is a function assigning to each point,  $P$  of  $D$  a vector in the plane with its initial point at  $P$ . "Intuitively, we can think of  $V$  as giving the velocity of some substance that is presently in  $D$ " (p. 33).

That is, we can think of affect relations as the "substance" being influenced, and the vector,  $V$ , as that which is giving those relations "velocity" or "impact." The vector is that which makes the affect relations effective.

The essential qualities of a vector are its length and direction. For our purposes, "length" can be defined as the "power" or "force" or "degree" of influence or effectiveness of the affect relation. "Direction" can be defined as the recipient of the affect.

The importance of vectors for intentional systems theory can be seen from the following description provided by Henle:

“Clearly the study of vector fields on a set  $D$  coincides with the study of continuous transformations of the set [that is, topology].

Vector fields have many important applications. The force fields arising from gravitation and electromagnetism are vector fields; the velocity vectors of a fluid in motion, such as the atmosphere (wind vectors), form a vector field; and gradients, such as the pressure gradient on a weather map or the height gradient on a relief chart, are vector fields. These examples are usually studied from the point of view of differential equations. A vector field,  $V(P) = (F(x,y), G(x,y))$ , determines a system of differential equations in the two unknowns  $x$  and  $y$ . These variables are taken to represent the position of a moving point in the plane dependent on a third variable, the time  $t$ . The system of differential equations takes the form:  $x' = F(x,y)$ ,  $y' = G(x,y)$ ; where the differentiation is with respect to  $t$ . Such a system is called **autonomous** because the right-hand sides are independent of time. A solution of this system consists of two functions expressing  $x$  and  $y$  in terms of  $t$ . These may be considered the parametric equations of a path in the plane: the path of a molecule of gas or liquid, the orbit of a planet or an electron, or the trajectory of a marble rolling down a hill, depending on the application. The original vector field  $V(P)$  gives the tangent vector to the path of motion at the point  $P = (x,y)$ .”

The “application” here is with respect to the affect relations of an intentional system. Such an application will be required if the full impact of topology is to be realized in this study. And, it will clearly be required in order to introduce the time element that is critical to any in-depth analysis of an intentional system.